环向离散加筋园柱曲板的侧压弹性稳定性

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引　言

为便于分析加筋壳的总体稳定性，经常可以把它简化为按正交各向异性的连续弹性体处理。但这种近似方法只能是在一定的条件下才能成立。通过分析周边滑动简支、具有环向离散加筋园柱曲板在侧压下的稳定性，在假设失稳前曲板为薄膜受力状态的前提下；本文比较了在不同参数范围内，按正交各向异性曲板计算与更精确地按环向离散加筋曲板计算的结果。从而对按正交各向异性曲板计算的适用条件提出一些看法。

目前虽有一些分析加筋园柱壳在侧压或全压作用下的稳定性的文章，例如参考文献〔1〕、〔2〕，但有关曲板的还很少。参考文献〔3〕、〔4〕分析了单层或正交各向异性曲板的稳定性。曲板的侧压弹性稳定实验的数据几乎空白。因此，为推进航空等有关事业的发展，加强曲板的实验与理论研究是极为必要的。

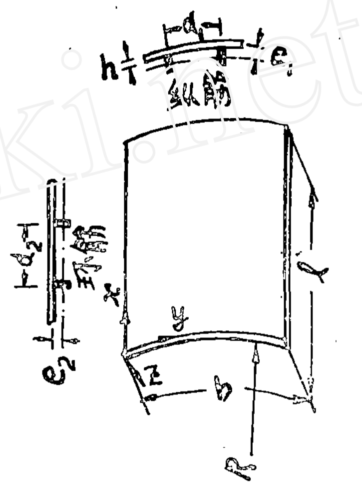

图　1

符　号

x、y、z　　座标，

h、b、l　　壳壁几何尺寸，

e_1、e_2　　纵、环筋的截面重心到壳壁中面距离，

R　　壳壁曲率半径，

d_1、d_2　　纵、环筋的间距，

A_X、A_Y　　纵、环筋的截面面积，

I_X、I_Y　　纵、环筋截面对壳壁中面的惯性矩，

u、v、w　　x、y、z　方向的位移，

ϵ_x、ϵ_y、ϵ_z　　应变，

σ_x、σ_y、τ_{xy}、τ_{yx}　　应力，

n_x、n_y、n_{xy}、n_{yx}　　壳壁内平面法向力、剪力，

m_x、m_y、m_{xy}、m_{yx}　　壳壁内弯矩、扭矩，

N_x、N_y　　纵，环筋的法向力（广义），

M_x、M_y　　纵、环筋的弯矩（广义），

$\widetilde{n}_x$、$\widetilde{n}_y$、$\widetilde{n}_{xy}$、$\widetilde{N}_x$、$\widetilde{N}_y$、　　失稳前壳壁，纵筋、环筋内给定的平面力，

$\overline{n}_x$、$\overline{n}_y$、$\overline{n}_{xy}$、$\overline{n}_{yx}$、$\overline{N}_x$、$\overline{N}_y$、边界上已知的壳壁、纵筋、环筋上的平面力，

$\overline{m}_x$、$\overline{m}_y$、$\overline{M}_x$、$\overline{M}_y$、　　边界上已知的壳壁，纵筋、环筋上的弯矩，

$\overline{v}_x$、$\overline{v}_y$、$\overline{V}_x$、$\overline{V}_y$、　　边界上已知的壳壁，纵筋、环筋上的等值剪力，

$\overline{R}$　　角点反力，P横向载荷，

Δ　　广义函数（即Dirac函数）的和（$\Delta=\sum\limits_i\delta(x-x_i)$ 其中δ为广义函数，i为环筋的序，x_i为各环筋的座标位置），

δ　　变分符号，

E、ν、E_x、ν_x、E_y、ν_y　　壳壁，纵筋、环筋的弹性模量及波桑系数，

D　　壳壁弯曲刚度 $\left(D=\dfrac{Eh^3}{12(1-\nu^2)}\right)$，

B　　壳壁平面刚度 $\left(B=\dfrac{Eh}{(1-\nu^2)}\right)$， N　环筋的数量

$\mu_1=\frac{E_xA_x(1-\nu^2)}{Ehd_1}$，　$\mu_2=\frac{E_yA_y(1-\nu^2)}{Ehd_2}$，

$\chi_1=\frac{E_xA_x(1-\nu^2)e_1}{Ehd_1\pi}\sqrt{\frac{B}{D}}$，

$\chi_2=\frac{E_yA_y(1-\nu^2)e_2}{Ehd_2\pi}\sqrt{\frac{B}{D}}$，

$\eta_1=\frac{E_xI_x}{Dd_1}$，　$\eta_2=\frac{E_yI_y}{Dd_2}$，

j、m、n　序数（其中m、n又可分别代表纵向、环向波数），

$\beta=\frac{b}{l}$（宽长比），

$K_r=\frac{b^2}{\pi^2 R}\sqrt{\frac{B}{D}}$（曲率参数），

$K_p=\frac{pRb^2}{\pi^2 D}$（临界载荷参数），

$K_A=\frac{A_y}{hd_2}$。

（一）基本方程

（1）基本假设：

①壳、筋截面在变形前、后保持为平面，

②壳、筋截面内应力各为直线分布，

③平面剪力扭矩仅由壳壁承受，

④在纵筋内$\sigma_y=0$，在环筋内$\sigma_x=0$，在壳、筋内的横向法应力σ_z均为0。

（2）应力与广义力的关系

在壳内：

$$\left.\begin{aligned}\sigma_x&=\frac{n_x}{h}+\frac{m_x}{h^3/12}Z,\\ \sigma_y&=\frac{n_y}{h}+\frac{m_y}{h^3/12}Z,\\ \tau_{xy}&=\frac{n_{xy}}{h}-\frac{m_{xy}}{h^3/12}Z,\\ \tau_{yx}&=\frac{n_{yx}}{h}+\frac{m_{yx}}{h^3/12}Z;\end{aligned}\right\}\quad(1)$$

其中　$n_{xy}=n_{yx}$，$m_{xy}=-m_{yx}$。

在纵筋内：

$$\sigma_x=\left(\frac{1}{I_x-A_xe_1^2}\right)\left[\left(\frac{I_x}{A_x}N_x+M_xe_1\right)+\left(M_x-N_xe_1\right)Z\right],\qquad(2)$$

在环筋内：

$$\sigma_y=\left(\frac{1}{I_y-A_ye_2^2}\right)\left[\left(\frac{I_y}{A_y}N_y-M_ye_2\right)+\left(M_y-N_ye_2\right)Z\right]。\qquad(3)$$

（3）应变与位移的关系

在壳内：

$$\left.\begin{aligned}\epsilon_x&=\frac{\partial u}{\partial x}-\frac{\partial^2 w}{\partial x^2}Z,\\ \epsilon_y&=\left(\frac{\partial v}{\partial y}-\frac{w}{R}\right)-\frac{\partial^2 w}{\partial y^2}Z,\\ \gamma_{xy}&=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}-2\frac{\partial^2 w}{\partial x\partial y}Z;\end{aligned}\right\}\quad(4)$$

在纵筋内：

$$\epsilon_x=\frac{\partial u}{\partial x}-\frac{\partial^2 w}{\partial x^2}Z;\qquad(5)$$

在环筋内：

$$\epsilon_y=\left(\frac{\partial v}{\partial y}-\frac{w}{R}\right)-\frac{\partial^2 w}{\partial y^2}Z。\qquad(6)$$

（4）Reissner广义变分原理〔5〕

在小挠度变形的情况下，应用Reissner的广义变分原理〔5〕可以推导出基本方程。变分原理的表达式是：

$$\delta\left[\iiint_V F\,dV-\iint_{S_\sigma}\left(\bar{p}_x u+\bar{p}_y v+\bar{p}_z w\right)dS_\sigma\right]=0;\qquad(7)$$

其中$F=\sigma_x\epsilon_x+\sigma_y\epsilon_y+\sigma_z\epsilon_z+\tau_{xy}\gamma_{xy}+\tau_{xz}\gamma_{xz}+\tau_{yz}\gamma_{yz}-W$，

W为以应力表示的应变能，

S_σ为已知应力的边界，

V为体积。

将纵筋按照应变能相等的原则折算到壳壁内，经过详细运算，从（1）—（6）式代入（7）式后可得到环向离散加筋园柱曲板的以下基本方程。

平衡方程：

$$\left.\begin{aligned}
&\frac{\partial}{\partial x}\left(n_x+\frac{N_x}{d_1}\right)+\frac{\partial}{\partial y}n_{xy}=0\ ,\\
&\frac{\partial}{\partial x}n_{xy}+\frac{\partial}{\partial y}\left(n_y+N_y\Delta\right)=0\ ,\\
&\frac{\partial^2}{\partial x^2}\left(m_x+\frac{Mx}{d_1}\right)+\frac{\partial^2}{\partial y^2}(m_y+M_y\Delta)-\frac{\partial^2}{\partial x\partial y}m_{xy}+\frac{\partial^2}{\partial x\partial y}m_{yx}+\frac{1}{R}(n_y+N_y\Delta)+\\
&+\left(\widetilde{n}_x+\frac{\widetilde{N}_x}{d_1}\right)\frac{\partial^2 w}{\partial x^2}+\left(\widetilde{n}_y+\widetilde{N}_y\Delta\right)\frac{\partial^2 w}{\partial y^2}+2\widetilde{n}_{xy}\frac{\partial^2 w}{\partial x\partial y}+P=0\ 。
\end{aligned}\right\}\quad(8)$$

广义力与广义位移的关系：

$$\left.\begin{aligned}
&n_x=B\left[\frac{\partial u}{\partial x}+\nu\left(\frac{\partial v}{\partial y}-\frac{w}{R}\right)\right],\\
&\frac{N_x}{d_1}=\frac{E_xA_x}{d_1}\left[\frac{\partial u}{\partial x}-\frac{\partial^2 w}{\partial x^2}e_1\right]\ ,\\
&n_y=B\left[\left(\frac{\partial v}{\partial y}-\frac{w}{R}\right)+\nu\frac{\partial u}{\partial x}\right],\\
&N_y=E_yA_y\left[\left(\frac{\partial v}{\partial y}-\frac{w}{R}\right)-\frac{\partial^2 w}{\partial y^2}e_2\right]\ ,\\
&n_{xy}=\frac{B(1-\nu)}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\ ,\\
&m_x=-D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)\ ,\\
&\frac{M_x}{d_1}=\frac{E_xI_x}{d_1}\left(-\frac{\partial^2 w}{\partial x^2}+\frac{A_xe_1}{I_x}\frac{\partial u}{\partial x}\right)\ ,\\
&m_y=-D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)\ ,\\
&M_y=E_yI_y\left[-\frac{\partial^2 w}{\partial y^2}+\frac{A_ye_2}{I_y}\left(\frac{\partial v}{\partial y}-\frac{w}{R}\right)\right]\ ,\\
&m_{xy}=-m_{yx}=D(1-\nu)\frac{\partial^2 w}{\partial x\partial y}\ 。
\end{aligned}\right\}\quad(9)$$

边界条件：

$$\left.\begin{aligned}
&\text{在曲边}\quad n_x+\frac{N_x}{d_1}=\overline{n}_x+\frac{\overline{M}_x}{d_1}\quad\text{或}u=0\ ,\\
&\qquad n_{xy}=\overline{n}_{xy}\quad\text{或}v=0\ ,\\
&\qquad m_y+\frac{M_x}{d_1}=\overline{m}_x+\frac{\overline{M}_x}{d_1}\text{或}\frac{\partial w}{\partial x}=0\ ,\\
&\frac{\partial}{\partial x}\left(m_x+\frac{M_x}{d_1}\right)+\frac{\partial m_{yx}}{\partial y}-\frac{\partial m_{xy}}{\partial y}+\left(\widetilde{n}_x+\frac{\widetilde{M}_x}{d_1}\right)\frac{\partial w}{\partial x}+\widetilde{n}_{xy}\frac{\partial w}{\partial y}=\left(\overline{v}_x+\frac{\overline{V}_x}{d_1}\right)\text{或}w=0;\\
&\text{在直边}\quad n_y+N_y\Delta=\overline{n}_y+\overline{N}_y\Delta\quad\text{或}v=0\ ,
\end{aligned}\right\}\quad(10)$$

$$n_{xy}=\overline{n}_{xy} \quad \text{或} u=0,$$

$$m_y+M_y\Delta=\overline{M}_y\Delta \quad \text{或} \frac{\partial w}{\partial y}=0,$$

$$\frac{\partial}{\partial y}\left(m_y+M_y\Delta\right)-\frac{\partial m_{xy}}{\partial x}+\frac{\partial m_{yx}}{\partial x}+\left(\tilde{n}_y+\tilde{M}_y\Delta\right)\frac{\partial w}{\partial y}+\tilde{n}_{xy}\frac{\partial w}{\partial x}=\left(\overline{v}_y+\overline{V}_y\Delta\right)$$

或 $w=0$；

在角点 $m_{xy}-m_{yx}=\overline{R}$ 或 $w=0$。

以上（9）式与文献〔1〕中的有关结果相类同。

（二）微分方程及其解法

假定失稳前曲板为薄膜受力状态，在受侧压时，曲板内的给定平面力可有以下两种写法：

（1）当横向载荷P为均分布时：

$$\left.\begin{aligned}&\tilde{n}_x+\frac{\tilde{N}x}{d_1}=\tilde{n}_{xy}=0,\\&\tilde{n}_y+\tilde{N}_y\Delta=-PR;\end{aligned}\right\} \tag{11}$$

这也就表明，壳壁内的平面法向力$\tilde{n}_y$是不连续的；在附有环筋处，壳壁内$\tilde{n}_y$的绝对值突然变小。为消除此突变，保持薄膜受力状态，则需在有环筋处附加环向线分布载荷，以使环筋内的予加应力与壳壁内的相同。

$$\left.\begin{aligned}&(2)\ \text{当}\ P=\overline{P}+\frac{\overline{P}A_y}{h}\Delta\text{时：}\\&\tilde{n}_x+\frac{\tilde{N}_x}{d_1}=\tilde{n}_{xy}=0,\\&\tilde{n}_y+\tilde{N}_y\Delta=-\overline{P}R\left(1+\frac{A_y}{h}\Delta\right)。\end{aligned}\right\} \tag{12}$$

以下将简称（11）、（12）式所代表的情况为情况（1）、情况（2）。显见情况（1）是情况（2）的特殊情况。当线分布载荷趋于零时，情况（2）即趋于情况（1）。为此，以下将以情况（2）为出发点推导微分方程。

将（9）式代入（8）式中的前三个，按照通常的作法〔6〕，在假定失稳前曲板为薄膜受力状态时，可以得到计算环向离散加筋园柱曲板的侧压稳定性的微分方程。

$$\left[(1+\mu_1)\frac{\partial^2}{\partial x^2}+\frac{1-\nu}{2}\frac{\partial^2}{\partial y^2}\right]u+\left[\frac{1+\nu}{2}\frac{\partial^2}{\partial x\partial y}\right]v-\left[\frac{\nu}{R}\frac{\partial}{\partial x}+\frac{\chi_1\pi}{\sqrt{B/D}}\frac{\partial^3}{\partial x^3}\right]w=0, \tag{13}$$

$$\left[\frac{1-\nu}{2}\frac{\partial^2}{\partial x\partial y}\right]u+\left[\frac{1-\nu}{2}\frac{\partial^2}{\partial x^2}+\left(1+\frac{E_yA_y(1-\nu^2)}{Eh}\Delta\right)\frac{\partial^2}{\partial y^2}\right]v-\left[\frac{1}{R}\left(1+\frac{E_yA_y(1-\nu^2)}{Eh}\Delta\right)\right.$$
$$\left.\frac{\partial}{\partial y}+\frac{E_yA_y(1-\nu^2)e_2}{Eh}\Delta\frac{\partial^3}{\partial y^3}\right]w=0, \tag{14}$$

$$\left[\frac{\nu}{R}\frac{\partial}{\partial x}+\frac{\chi_1\pi}{\sqrt{B/D}}\frac{\partial^3}{\partial x^3}\right]u+\left[\frac{1}{R}\left(1+\frac{E_yA_y(1-\nu^2)}{Eh}\Delta\right)\frac{\partial}{\partial y}+\frac{E_yA_y(1-\nu^2)e_2}{Eh}\Delta\right)\frac{\partial^3}{\partial y^3}\Big]v-$$
$$-\frac{D}{B}\left[(1+\eta_1)\frac{\partial^4}{\partial x^4}+2\frac{\partial^4}{\partial x^2\partial y^2}+\left(1+\frac{E_yI_y}{D}\Delta\right)\frac{\partial^4}{\partial y^4}+\left(\frac{\overline{p}R}{D}\left(1+\frac{A_y}{h}\Delta\right)+\right.\right.$$

$$+\frac{2E_yA_ye_2}{RD}\Delta\Big)\frac{\partial^2}{\partial y^2}+\frac{B}{R^2D}\Big(1+\frac{E_yA_y(1-\nu^2)}{Eh}\Delta\Big)\Big]w=0。\tag{15}$$

令

$$\left.\begin{aligned}u&=\sum_{mn}a_{mn}u_{mn},\\ v&=\sum_{m,n}b_{mn}v_{mn},\\ w&=\sum_{m,n}c_{mn}w_{mn};\end{aligned}\right\}\tag{16}$$

其中u_{mn}、v_{mn}、w_{mn}为满足所有边界条件的位移函数。于是由伽辽金法可以对应（13）、（14）、（15）各式得到

$$\int\int\{\cdots\cdots\}u_{mn}dxdy=0,\tag{13)'$$

$$\int\int\{\cdots\cdots\}v_{mn}dx\,dy=0,\tag{14)'$$

$$\int\int\{\cdots\cdots\}w_{mn}dx\,dy=0。\tag{15)'$$

（三）滑动简支条件下的稳定方程及其化简

$$\left.\begin{aligned}&\text{此时，在曲边上：}\quad N_x=0,\ v=0,\\ &\qquad\qquad\qquad\quad M_x=0,\ w=0,\\ &\text{在直边上：}\quad N_y=0,\ u=0,\\ &\qquad\qquad\qquad\quad M_y=0,\ w=0,\end{aligned}\right\}\tag{17}$$

令

$$\left.\begin{aligned}u&=\sum_{m,n}a_{mn}\cos\frac{m\pi x}{l}\sin\frac{n\pi y}{b},\\ v&=\sum_{m,n}b_{mn}\sin\frac{m\pi x}{l}\cos\frac{n\pi y}{b},\\ w&=\sum_{m,n}c_{mn}\sin\frac{m\pi x}{l}\sin\frac{n\pi y}{b}。\end{aligned}\right\}\tag{18}$$

为便于计算，取环向加筋为等间距的情况，则将（18）代入（13）′、（14）′、（15）′，即得计算稳定性的代数方程组：

$$\left\{1+\frac{1-\nu}{2}\frac{m^2\beta^2}{n^2}-\frac{\left(\frac{1+\nu}{2}\right)^2m^2\beta^2}{(1+\mu_1)m^2\beta^2+\frac{1-\nu}{2}n^2}\right\}b_{mn}'+\mu_2\sum_j\delta_{mj}\,b_{jn}'+\frac{1}{n}\Big\{K_r+$$

$$+\frac{\left(\frac{1+\nu}{2}m^2\beta^2\right)\left(\pi\chi_1 m^2\beta^2-\nu K_r\right)}{(1+\mu_1)m^2\beta^2+\left(\frac{1-\nu}{2}\right)n^2}\Bigg]c_{mn}+\frac{1}{n}(\mu_2K_r-\pi\chi_2n^2)\sum_j\delta_{mj}c_{jn}=0, \tag{19}$$

$$\frac{1}{n}\left\{K_r+\frac{\left(\frac{1+\nu}{2}m^2\beta^2\right)\left(\pi\chi_1m^2\beta^2-\nu K_r\right)}{(1+\mu_1)m^2\beta^2+\left(\frac{1-\nu}{2}\right)n^2}\right\}b_{mn}'+\frac{1}{n}(\mu_2K_r-\pi\chi_2n^2)\sum_j\delta_{mj}b_{jn}'+\frac{1}{n^2}\left[(1+\eta_1)m^4\beta^4\right.$$

$$\left.+2m^2n^2\beta^2+n^4+K_r^2-\frac{m^2\beta^2(\pi\chi_1m^2\beta^2-\nu K_r)^2}{(1+\mu_1)m^2\beta^2+\frac{1-\nu}{2}n^2}\right]c_{mn}+\left[\eta_2n^2-2\pi\chi_2K_r+\frac{1}{n^2}\mu_2K_r^2\right]$$

$$\sum_j\delta_{mj}c_{jn}-\left(\frac{1}{1+\frac{N}{N+1}K_A}\right)K_Pc_{mn}-\left(\frac{K_A}{1+\frac{N}{N-1}K_A}\right)K_P\sum_j\delta_{mj}c_{jn}=0; \tag{20}$$

其中 $b_{mn}'=\frac{b}{\pi}\sqrt{\frac{B}{D}}b_{mn}$,

$$\delta_{mj}=\begin{cases}1 & 当\ (j-m)=2k(N+1)\ 时k=0、1、2\cdots\\-1 & 当\ (j+m)=2k(N+1)\ 时k=1、2、3\cdots\\0 & 其他\end{cases}$$

在(20)式中已考虑了使前节中情况(1),情况(2)的总载荷量相等,以便于计算比较(此时 $\overline{P}=\left(\frac{1}{1+\frac{N}{N+1}K_A}\right)P$),如令 $K_A=0$ 即为情况(1),否则为情况(2)。

以上(10)、(20)式可缩写为矩阵向量型式:

$$\left.\begin{aligned}&[V_1]\overline{v}+[W_1]\overline{w}=0,\\&[V_2]\overline{v}+[W_2]\overline{w}=K_P[P]\overline{w};\end{aligned}\right\} \tag{21}$$

其中

$$\overline{V}=\begin{pmatrix}b_{1n}\\b_{2n}\\b_{3n}\\\vdots\end{pmatrix},\quad \overline{w}=\begin{pmatrix}c_{1n}\\c_{2n}\\c_{3n}\\\cdots\end{pmatrix};$$

又 $[V_1]$、$[V_2]$、$[W_1]$、$[W_2]$、$[P]$ 为(19)、(20)式中有关的系数所组成的矩阵。于是可归并为:

$$[P]^{-1}\{[W_2]-[V_2][V_1]^{-1}[W_1]\}\overline{w}=K_P\overline{w}. \tag{22}$$

以下的问题就是求解(22)式的最小特征值 K_P 及相应的特征向量 $\overline{w}$。

在参考文献〔6〕中给出了使类似于(22)式中的矩阵阵价的办法,最后可简化为计算(N+1)个子矩阵问题,从中可以即刻得到

(1)加筋曲板按正交各向异性理论计算的总体失稳的公式为:

$$K_P=\frac{1}{n^2}\left\{\left[(1+\eta_1)m^4\beta^4+2m^2n^2\beta^2+(1+\eta_2)n^4\right]-\right.$$

$$-\frac{\pi^2}{\frac{1-\nu}{2}(1+\mu_1)m^4\beta^4+\left[(1+\mu_1)(1+\mu_2)-\nu\right]m^2n^2\beta^2+\frac{1-\nu}{2}(1+\mu_2)n^4}\left[\frac{1-\nu}{2}\chi_1^2m^8\beta^8+\right.$$

$$+(1+\mu_2)\chi_1{}^2m^6n^2\beta^6-\nu(1-\nu)\frac{\chi_1K_r}{\pi}m^6\beta^6-(1+\nu)\chi_1\chi_2m^4n^4\beta^4+(1-\nu)\frac{K_r}{\pi}\Big((1+\mu_1)\chi_2+$$

$$+(1+\mu_2)\chi_1\Big)m^4n^2\beta^4-\frac{1-\nu}{2}\Big((1+\mu_1)(1+\mu_2)-\nu^2\Big)\frac{K_r{}^2}{\pi^2}m^4\beta^4-\nu(1-\nu)\frac{\chi_2K_r}{\pi}m^2n^4\beta^2+$$

$$+(1+\mu_1)\chi_2{}^2m^2n^6\beta^2+\frac{1-\nu}{2}\chi_2{}^2n^8\Big]\Big\} \tag{23}$$

（2）在环筋之间的局部失稳的公式为：

$$K_P=\frac{1}{n^2}\Big\{\Big[(1+\eta_1)m^4\beta^4+2m^2n^2\beta^2+n^4\Big]-$$

$$-\frac{\pi^2}{\left(\frac{1-\nu}{2}\right)(1+\mu_1)m^4\beta^4+(1+\mu_1-\nu)m^2n^2\beta^2+\frac{1-\nu}{2}n^4}\Big[\frac{1-\nu}{2}\chi_1{}^2m^8\beta^8+$$

$$+\chi_1{}^2m^6n^2\beta^6-\nu(1-\nu)\frac{\chi_1}{\pi}K_rm^6\beta^6+(1-\nu)\frac{\chi_1K_r}{\pi}m^4n^2\beta^4-\frac{1-\nu}{2}(1+\mu_1-\nu^2)$$

$$\frac{K_r{}^2}{\pi^2}m^4\beta^4\Big]\Big\} \tag{24}$$

（四）计算结果与分析

下面以周边滑动简支、受均布侧压为例，分析具有环向离散加筋园柱曲板的弹性稳定性。经过计算环筋之间壳壁的局部失稳载荷，并将按环向离散方法与按正交各向异性方法总体失稳载荷作了比较之后，在所给参数范围内，可以得出以下看法。

（1）按照情况（1）与情况（2）两种情况的计算比较说明：在总载荷量相等的条件下，在壳壁局部失稳之前，二者差别不大。在按照《表1》所给的局部失稳载荷以后，《表2》中的K_P值为《表1》中的$\left(1+\frac{N}{N+1}K_A\right)$倍。原因是，按情况（2）计算时，当环筋间的均布载荷达到使壳壁局部失稳时，连同环筋上的线分布载荷，其总载荷量恰为在情况（1）下局部失稳时的总载荷量的$\left(1+\frac{N}{N+1}K_A\right)$倍。从而使离散方法计算的总体失稳载荷也提高$\left(1+\frac{N}{N+1}K_A\right)$倍。为简便起见，以下均按情况（1）作比较分析。

（2）考虑环筋法向力的作用对计算总体失稳的影响极微。比较《表1》、《表3》可见，其差别仅为1％左右。

（3）由《表4》、《表5》可见，一般地说，环筋在曲板的内侧（χ_2为正）较在外侧（χ_2为负）为佳。

（4）比较《表4》、《表6》、《表7》、《表8》可说，曲率参数的影响较大。K_r愈大，承载能力愈高。

（5）综合以上各表，可归纳出以下一点，即局部失稳的载荷可作为按正交各向异性方法与按离散方法计算总体失稳的分界限。在此之前二者是一致的。

（6）加纵筋后，对提高总体失稳载荷的作用虽不大但由于它提高了局部失稳的载荷，从而扩大了按正交各向异性方法处理问题的适用范围。（见《表8》、《表10》）。

（7）随着宽长比β的增大，K_P值也不断提高。（见表（11））

以上各条看法中，第5条是最主要的。也就回答了本文一开始所提出的问题。

《表1》$K_r=1.5\times10^2$ $\beta=1$

η_2 \ N	1	2	3	4	5	∞
1	36.20(1)(4)	37.46(1)(4)	37.83(1)(4)	38.02(1)(4)	38.14(1)(4)	38.36(1)(4)
2	46.48(1)(4)	50.66(1)(3)	50.86(1)(3)	50.98(1)(3)	51.06(1)(3)	51.26(1)(3)
5	55.02($^{1,}_{3}$)(6)	74.87(1)(3)	76.05(1)(3)	76.75(1)(3)	77.22(1)(3)	78.22(1)(3)
10	55.53($^{1,}_{3}$)(6)	82.68($^{2,}_{4}$)(7)	114.5 ($^{3,}_{5}$)(7)	117.4 (1)(3)	119.2 (1)(3)	123.22(1)(3)
20	55.79($^{1,}_{3}$)(6)	83.01($^{2,}_{4}$)(7)	115.3 ($^{3,}_{5}$)(7)	152.3 ($^{4,}_{6}$)(8)	196.1 ($^{5,}_{7}$)(9)	213.2(1)(3)
50	55.95($^{1,}_{3}$)(6)	83.20($^{2,}_{4}$)(7)	115.8 ($^{3,}_{5}$)(7)	152.7 ($^{4,}_{6}$)(8)	196.5 ($^{5,}_{7}$)(9)	405.2(1)(2)
100	56.00($^{1,}_{3}$)(6)	83.27($^{2,}_{4}$)(7)	116.0 ($^{3,}_{5}$)(7)	152.8 ($^{4,}_{6}$)(8)	196.6 ($^{5,}_{7}$)(9)	605.2(1)(2)
200	56.03($^{1,}_{3}$)(6)	83.20 ($^{2,}_{4}$)(7)	116.0 ($^{3,}_{5}$)(7)	152.9 ($^{4,}_{6}$)(8)	196.7 ($^{5,}_{7}$)(9)	1005 (1)(2)
400	56.04($^{1,}_{3}$)(6)	83.32 ($^{2,}_{4}$)(7)	116.1 ($^{3,}_{5}$)(7)	152.9 ($^{4,}_{6}$)(8)	196.7 ($^{5,}_{7}$)(9)	1805 (1)(2)
1000	56.05($^{1,}_{3}$)(6)	83.33 ($^{2,}_{4}$)(7)	116.1 ($^{3,}_{5}$)(7)	152.9 ($^{4,}_{9}$)(8)	196.7 ($^{5,}_{7}$)(9)	4205 (1)(2)
	48.78(2)(5)	78.36 (3)(6)	110.8 (4)(7)	148.2 (5)(8)	192.2 (6)(9)	

《表2》 $K_r=1.5\times10^2$ $\beta=1$ $K_A=0.45$

η_2 \ N	1	2	3	4	5
1	32.27(1)(4)	34.33(1)(4)	35.35(1)(4)	35.95(1)(4)	36.35(1)(4)
2	43.32(1)(3)	46.02(1)(3)	47.34(1)(3)	48.18(1)(3)	48.71(1)(3)
5	64.63(1)(3)	69.45(1)(3)	71.70(1)(3)	73.05(1)(3)	73.96(1)(3)
10	67.78($^{1,}_{3}$)(6)	106.3 (1)(3)	110.9 (1)(3)	113.7 (1)(3)	115.5(1)(3)
20	68.33($^{1,}_{3}$)(6)	107.8 ($^{2,}_{4}$)(3)	154.2 ($^{3,}_{5}$)(7)	190.9 (1)(3)	195.5(1)(3)
50	68.53($^{1,}_{3}$)(6)	108.1 ($^{2,}_{4}$)(7)	154.8 ($^{3,}_{5}$)(7)	207.6 ($^{4,}_{6}$)(8)	270.1($^{5,}_{7}$)(9)
100	68.60($^{1,}_{3}$)(6)	108.2 ($^{2,}_{4}$)(7)	155.1 ($^{3,}_{5}$)(7)	207.9 ($^{4,}_{6}$)(8)	270.4($^{5,}_{7}$)(9)
200	68.63($^{1,}_{3}$)(6)	108.2 ($^{2,}_{4}$)(7)	155.1 ($^{3,}_{5}$)(7)	207.9 ($^{4,}_{6}$)(8)	270.4($^{5,}_{7}$)(9)
400	68.64($^{1,}_{3}$)(6)	108.3 ($^{2,}_{4}$)(7)	155.2 ($^{3,}_{5}$)(7)	207.9 ($^{4,}_{6}$)(8)	270.4($^{5,}_{7}$)(9)
1000	68.66($^{1,}_{3}$)(6)	108.3 ($^{2,}_{4}$)(7)	155.2 ($^{3,}_{5}$)(7)	207.9 ($^{4,}_{6}$)(8)	270.4($^{5,}_{7}$)(9)

《表 3 》$K_r=1.5\times10^2$　$\beta=1$　$\mu_2=0.5$

η_2 \ N	1	2	3	4	5	∝
1	36.53(1)(4)	37.83(1)(4)	38.15(1)(4)	38.28(1)(4)	38.35(1)(4)	38.47(1)(4)
2	47.21(1)(4)	51.30(1)(3)	51.43(1))3)	51.48(1)(3)	51.51(1)(3)	51.56(1)(3)
5	55.08$(^{1,}_{3})$(6)	76.69(1)(3)	77.51(1)(3)	77.89(1)(3)	78.11(1)(3)	78.56(1)(3)
10	55.59$(^{1,}_{3})$(6)	82.82$(^{2,}_{4})$(7)	115.0$(^{3,}_{5})$(7)	120.1(1)(3)	121.2(1)(3)	123.5 (1)(3)
20	55.85$(^{1,}_{3})$(6)	83.15$(^{2,}_{4})$(7)	115.8$(^{3,}_{5})$(7)	152.6$(^{4,}_{6})$(8)	196.3$(^{5,}_{7})$(9)	213.5 (1)(3)
50	56.01$(^{1,}_{1})$(6)	83.84$(^{2,}_{4})$(7)	116.2$(^{3,}_{5})$(7)	153.0$(^{4,}_{6})$(8)	196.7$(^{5,}_{7})$(9)	405.6 (1)(2)
100	56.06$(^{1,}_{3})$(6)	83.41$(^{2,}_{4})$(7)	116.3$(^{3,}_{5})$(7)	153.1$(^{4,}_{6})$(8)	196.8$(^{5,}_{7})$(9)	605.6 (1)(2)
200	56.09$(^{1,}_{3})$(6)	83.41$(^{2,}_{4})$(7)	116.5$(^{3,}_{5})$(7)	153.2$(^{4,}_{6})$(8)	196.9$(^{5,}_{7})$(9)	1005 (1)(2)
400	56.10$(^{1,}_{3})$(9)	83.46$(^{2,}_{4})$(7)	116.5$(^{3,}_{5})$(7)	153.2$(^{4,}_{6})$(8)	196.9$(^{5,}_{7})$(9)	1805 (1)(2)
1000	56.11$(^{1,}_{3})$(6)	83.47$(^{2,}_{4})$(7)	116.6$(^{3,}_{5})$(7)	153.2$(^{4,}_{6})$(8)	197.0$(^{3,}_{7})$(9)	4205 (1)(2)
局部失稳	48.78(4)(5)	78.36(3)(6)	110.8(4)(7)	148.2(5)(8)	192.2(6)(9)	

《表 4 》$K_r=1.5\times10^2$　$\beta=1$　$\mu_2=0.5$　$\chi_2=+0.35$

η_2 \ N	1	2	3	4	5	∝
2	28.69(1)(2)	30.58(1)(4)	52.54(1)(4)	34.41(1)(4)	36.14(1)(4)	44.44(1)(4)
5	54.81$(^{1,}_{3})$(5)	68.59(1)(3)	69.84(1)(3)	70.78(1)(3)	71.61(1)(3)	75.33(1)(3)
10	55.59$(^{1,}_{3})$(6)	82.83$(^{2,}_{4})$(7)	115.2 $(^{3,}_{5})$(7)	117.1 (1)(3)	117.9 (1)(3)	120.3 (1)(3)
20	55.87$(^{1,}_{3})$(6)	83.20$(^{2,}_{4})$(7)	115.9 $(^{3,}_{5})$(7)	152.7 $(^{4,}_{6})$(8)	196.4 $(^{5,}_{7})$(9)	210.3 (1)(3)
50	56.02$(^{1,}_{3})$(6)	83.37$(^{2,}_{4})$(7)	116.3 $(^{3,}_{5})$(7)	153.1 $(^{4,}_{6})$(8)	196.8 $(^{5,}_{7})$(9)	405.1 (1)(2)
100	56.07$(^{1,}_{3})$(6)	83.42$(^{2,}_{4})$(7)	116.5 $(^{3,}_{5})$(7)	153.2 $(^{4,}_{6})$(8)	196.3 $(^{5,}_{7})$(9)	605.1 (1)(2)
200	56.09$(^{1,}_{3})$(6)	83.45$(^{2,}_{4})$(7)	116.5 $(^{3,}_{5})$(7)	153.2 $(^{4,}_{6})$(8)	196.9 $(^{5,}_{7})$(9)	1005.1 (1)(2)
400	56.10$(^{1,}_{3})$(6)	83.46$(^{2,}_{4})$(7)	116.5 $(^{3,}_{5})$(7)	153.3 $(^{4,}_{6})$(8)	197.0 $(^{5,}_{7})$(9)	1805 (1)(2)
1000	56.11$(^{1,}_{3})$(6)	83.47$(^{2,}_{4})$(7)	116.6 $(^{3,}_{5})$(7)	153.3 $(^{4,}_{6})$(8)	197.0 $(^{5,}_{7})$(9)	4205 (1))2)
局部失稳	48.78(2)(5)	78.35(3)(6)	110.8 (4)(7)	148.2 (5)(8)	192.2 (6)(9)	

《表5》 $K_r = 1.5\times10^2$ $\beta=1$ $\mu_2=0.5$ $\chi_2=-0.35$

η_2 \ N	1	2	3	4	5	∝
2	24.59(1)(4)	25.55(1)(4)	26.83(1)(4)	28.21(1)(4)	29.65(1)(4)	37.80(1)(4)
5	52.18(1)(5)	57.86(1)(3)	59.09(1)(3)	60.25(1)(3)	61.40(1)(3)	66.51(1)(3)
10	55.37($^{1,}_{3}$)(6)	82.48($^{2,}_{4}$)(7)	97.49(1)(3)	100.0 (1)(3)	102.3 (1)(3)	111.5 (1)(3)
20	55.77($^{1,}_{3}$)(6)	83.02($^{2,}_{4}$)(7)	115.4 ($^{3,}_{5}$)(7)	152.3 ($^{4,}_{6}$)(8)	180.2 (1)(3)	201.5 (1)(3)
50	55.98($^{1,}_{3}$)(6)	83.30($^{2,}_{4}$)(7)	116.1 ($^{3,}_{5}$)(7)	152.9 ($^{4,}_{6}$)(8)	196.6 ($^{5,}_{7}$)(9)	399.0 (1)(3)
100	56.05($^{1,}_{3}$)(6)	83.39($^{2,}_{4}$)(7)	116.4 ($^{3,}_{5}$)(7)	153.1 ($^{4,}_{6}$)(8)	196.8 ($^{5,}_{7}$)(9)	599.0 (1)(2)
200	56.08($^{1,}_{3}$)(6)	83.43($^{2,}_{4}$)(7)	116.5 ($^{3,}_{5}$)(7)	153.2 ($^{4,}_{6}$)(8)	196.9 ($^{5,}_{7}$)(9)	999.0 (1)(2)
400	56.10($^{1,}_{3}$)(6)	83.46($^{2,}_{4}$)(7)	116.5 ($^{3,}_{5}$)(7)	153.2 ($^{4,}_{6}$))8)	196.9 ($^{5,}_{7}$)(9)	1799 (1)(2)
1000	56.11($^{1,}_{3}$)(3)	83.47($^{2,}_{4}$)(7)	116.6 ($^{3,}_{5}$)(7)	153.3 ($^{4,}_{6}$)(8)	197.0 ($^{5,}_{7}$)(9)	4199 (1)(2)
局部失稳	48.78(2)(5)	78.36(3)(6)	110.8 (4)(7)	148.2 (5)(8)	192.2 (6)(9)	

《表6》 $K_r = 0.75\times10^2$ $\beta=1$ $\mu_2=0.5$ $\chi_2=+0.35$

η_2 \ N	1	2	3	4	5	∝
2	21.20(1)(3)	22.78(1)(3)	24.20(1)(3)	25.43(1)(3)	26.11(1)(3)	29.28(1)(3)
5	40.33(1)(4)	50.48(1)(3)	52.50(1)(3)	53.53(1)(3)	54.19(1)(3)	56.28(1)(3)
10	41.39($^{1,}_{3}$)(5)	64.29($^{2,}_{4}$)(5)	91.81($^{3,}_{5}$)(6)	93.37(1)(2)	93.57(1)(2)	94.12(1)(2)
20	41.65($^{1,}_{3}$)(5)	65.13($^{2,}_{4}$)(5)	92.52($^{3,}_{5}$)(6)	127.5 ($^{4,}_{6}$)(7)	133.4 (1)(2)	134.1 (1)(2)
50	41.78($^{1,}_{3}$)(5)	65.57($^{2,}_{4}$)(5)	92.89($^{3,}_{5}$)(6)	127.9 ($^{4,}_{6}$)(7)	171.0 ($^{5,}_{7}$)(8)	254.1 (1)(2)
100	41.82($^{1,}_{3}$)(5)	65.66($^{2,}_{4}$)(6)	93.01($^{3,}_{5}$)(6)	128.0 ($^{4,}_{6}$)(7)	171.1 ($^{5,}_{7}$)(8)	454.1 (1)(2)
200	41.84($^{1,}_{5}$)(5)	65.69($^{2,}_{4}$)(6)	93.06($^{3,}_{5}$)(6)	128.1 ($^{4,}_{6}$)(7)	171.1 ($^{5,}_{7}$)(8)	851.1 (1)(2)
400	41.85($^{1,}_{3}$)(5)	65.70($^{2,}_{4}$)(6)	93.09($^{3,}_{5}$)(6)	128.1 ($^{4,}_{6}$)(7)	171.2 ($^{5,}_{7}$)(8)	1654 (1)(2)
1000	41.86($^{1,}_{3}$)(5)	65.71($^{2,}_{4}$)(6)	93.11($^{3,}_{5}$)(6)	128.1 ($^{4,}_{6}$)(7)	171.2 ($^{5,}_{7}$)(8)	2279 (1)(1)
局部失稳	37.42(2)(5)	60.18(3)(5)	88.19(4)(6)	123.3 (5)(7)	165.6 (6)(7)	

《表7》$K_r = 2.5\times10^2$　$\beta = 1$　$\mu_2 = 0.5$　$\chi_2 = +0.35$

η_2 \ N	1	2	3	4	5	∞
2	36.81(1)(4)	37.96(1)(4)	39.52(1)(4)	41.22(1)(4)	42.88(1)(4)	54.50(1)(4)
5	68.98$(^{1,}_{3})$(6)	88.98(1)(4)	91.68(1)(4)	93.34(1)(4)	94.62(1)(4)	102.5 (1)(4)
10	70.74$(^{1,}_{3})$(7)	102.1 $(^{2,}_{4})$(8)	139.3 $(^{3,}_{5})$(9)	157.9 (1)(3)	158.6 (1)(3)	163.1 (1)(3)
20	71.03$(^{1,}_{3})$(7)	102.5 $(^{2,}_{4})$(8)	139.7 $(^{3,}_{5})$(9)	181.8 $(^{4,}_{6})$(9)	228.2$(^{5,}_{7})$(10)	253.1(1)(3)
50	71.18$(^{1,}_{7})$(7)	102.6 $(^{2,}_{4})$(8)	139.9 $(^{3,}_{5})$(9)	182.2 $(^{4,}_{6})$(9)	228.6$(^{5,}_{7})$(10)	523.1(1)(3)
100	71.23$(^{1,}_{3})$(7)	102.7 $(^{2,}_{4})$(8)	139.9 $(^{3,}_{5})$(9)	182.4 $(^{4,}_{6})$(9)	228.7$(^{5,}_{7})$(10)	961.6(1)(2)
200	71.25$(^{1,}_{3})$(7)	102.7 $(^{2,}_{4})$(8)	140.0 $(^{3,}_{5})$(9)	182.4 $(^{4,}_{6})$(9)	228.7$(^{5,}_{7})$(10)	1361 (1)(2)
400	71.26$(^{1,}_{3})$(7)	102.7 $(^{2,}_{4})$(8)	140.0 $(^{3,}_{5})$(9)	182.4 $(^{4,}_{6})$(9)	228.7$(^{5,}_{7})$(10)	2161 (1)(2)
1000	71.27$(^{1,}_{3})$(7)	102.8 $(^{2,}_{4})$(8)	140.0 $(^{3,}_{5})$(9)	182.5 $(^{4,}_{6})$(9)	228.8$(^{5,}_{7})$(10)	4561 (1)(2)
局部失稳	59.79(2)(6)	95.81 (3)(7)	134.5 (4)(8)	176.6 (5)(9)	223.6(6)(10)	

《表8》　$K_r = 5.0\times10^2$　$\beta = 1$　$\mu_2 = 0.5$　$\chi_2 = +0.35$

η_2 \ N	1	2	3	4	5	∞
2	49.70(1)(5)	50.56 (1)(5)	51.70(1)(5)	53.28(1)(5)	55.11(1)(5)	77.92(1)(5)
5	95.37$(^{1,}_{3})$(8)	126.6 (1)(4)	127.8 (1)(4)	129.0 (1)(4)	130.2 (1)(4)	144.8 (1)(4)
10	96.96$(^{1,}_{3})$(8)	138.2 $(^{2,}_{4})$(9)	185.5$(^{3,}_{5})$(10)	213.2 (1)(4)	214.4 (1)(4)	224.8 (1)(4)
20	97.56$(^{1,}_{3})$(8)	139.0 $(^{2,}_{4})$(9)	185.7$(^{3,}_{5})$(10)	236.9$(^{4,}_{6})$(11)	292.3$(^{5,}_{7})$(12)	384.8 (1)(4)
50	97.87$(^{1,}_{3})$(8)	139.5$(^{2,}_{4})$(9)	186.1$(^{3,}_{5})$(10)	237.3$(^{4,}_{6})$(11)	292.6$(^{5,}_{7})$(12)	717.6 (1)(3)
100	97.97$(^{1,}_{3})$(8)	139.4$(^{2,}_{4})$(9)	186.3$(^{3,}_{5})$(10)	237.3$(^{4,}_{6})$(11)	292.7$(^{5,}_{7})$(12)	1167 (1)(3)
200	98.01$(^{1,}_{3})$(8)	139.6$(^{2,}_{4})$(9)	186.3$(^{3,}_{5})$(10)	237.4$(^{4,}_{6})$(11)	292.8$(^{5,}_{7})$(12)	2067 (1)(3)
400	98.04$(^{1,}_{3})$(8)	139.6$(^{2,}_{4})$(9)	186.4$(^{3,}_{5})$(10)	237.5$(^{4,}_{6})$(11)	292.8$(^{5,}_{7})$(12)	3828 (1)(2)
1000	98.05$(^{1,}_{3})$(3)	139.6$(^{2,}_{4})$(9)	186.4$(^{3,}_{5})$(10)	237.5$(^{4,}_{6})$(11)	292.8$(^{5,}_{7})$(12)	6228 (1)(2)
局部失稳	83.02(2)(7)	127.2(3)(9)	176.6(4)(10)	229.7(5)(11)	286.4(6)(14)	

《表 9》 $K_r=1.5\times10^2$ $\beta=1$ $\mu_2=0.5$ $\chi_2=0.35$ $\eta_1=1.00\times10^2$

η_2 \ N	1	2	3	4	5	∞
2	37.58(1)(4)	40.26(1)(4)	42.65(1)(4)	44.58(1)(4)	46.04(1)(4)	51.50(1)(4)
5	81.60(1)(4)	85.06(1)(3)	86.58(1)(3)	87.84(1)(3)	88.37(1)(3)	90.61(1)(3)
10	124.9 (1)(3)	129.8 (1)(3)	131.6 (1)(3)	132.6 (1)(3)	133.4 (1)(3)	135.6(1)(3)
20	135.5 ($\frac{1,}{3}$)(8)	217.6 (1)(3)	221.0 (1)(3)	222.5 (1)(3)	223.3 (1)(3)	225.6(1)(3)
50	137.0 ($\frac{1,}{3}$)(8)	252.0($\frac{2,}{4}$)(11)	408.8($\frac{3,}{5}$)(14)	457.8(1)(2)	458.0(1)(2)	458.5(1)(2)
100	137.4 ($\frac{1,}{3}$)(8)	252.6($\frac{2,}{4}$)(11)	409.5($\frac{3,}{5}$)(14)	609.5($\frac{4,}{6}$)(17)	657.7(1)(2)	658.5(1)(2)
200	137.6 ($\frac{1,}{3}$)(8)	252.8($\frac{2,}{4}$)(11)	409.8($\frac{3,}{5}$)(14)	609.9($\frac{4,}{6}$)(17)	853.5($\frac{5,}{7}$)(20)	1058 (1)(2)
400	137.7 ($\frac{1,}{3}$)(8)	253.0($\frac{2,}{4}$)(11)	410.0($\frac{3,}{5}$)(14)	610.0($\frac{4,}{6}$)(17)	853.7($\frac{5,}{7}$)(20)	1858 (1)(2)
1000	137.8 ($\frac{1,}{3}$)(8)	253.0($\frac{2,}{4}$)(11)	410.1($\frac{3,}{5}$)(14)	610.1($\frac{4,}{6}$)(17)	853.8($\frac{5,}{7}$)(20)	4258 (1)(2)
局部失稳	92.88(2)(7)	201.5(3)(11)	355.0(4)(13)	553.3(5)(16)	796.1(6)(19)	

《表10》 $K_r=1.5\times10^2$ $\beta=1$ $\mu_2=0.5$ $\chi_2=+0.35$ $\eta_2=2.00\times10^2$

η_2 \ N	1	2	3	4	5	∞
2	43.88(1)(4)	46.56(1)(4)	48.95(1)(4)	50.86(1)(4)	52.32(1)(4)	57.75 (1)(4)
5	87.57(1)(4)	94.43(1)(4)	96.99(1)(4)	98.77(1)(3)	99.49(1)(3)	101.7(1)(3)
10	137.2 (1)(3)	141.0(1)(3)	142.7(1)(3)	143.7(1)(3)	144.5(1)(3)	146.7(1)(3)
20	185.0($\frac{1,}{3}$)(9)	229.8(1)(3)	232.4(1)(3)	233.7(1)(3)	234.4(1)(3)	236.7(1)(3)
50	187.5($\frac{1,}{3}$)(9)	346.0($\frac{2,}{4}$)(13)	482.5(1)(2)	482.8(1)(2)	483.0(1)(2)	483.5(1)(2)
100	188.5($\frac{1,}{3}$)(9)	346.8($\frac{2,}{4}$)(13)	562.7($\frac{3,}{5}$)(16)	682.5(1)(2)	682.9(1)(2)	683.5(1)(2)
200	188.6($\frac{1,}{3}$)(10)	347.1($\frac{2,}{4}$)(13)	563.3($\frac{3,}{5}$)(16)	838.1($\frac{4,}{6}$)(20)	1082 (1)(2)	1083 (1)(2)
400	188.8($\frac{1,}{3}$)(10)	347.3($\frac{2,}{4}$)(13)	563.5($\frac{3,}{5}$)(16)	838.4($\frac{4,}{6}$)(20)	1174 ($\frac{5,}{7}$)(23)	1883 (1)(2)
1000	188.8($\frac{1,}{3}$)(10)	347.4($\frac{2,}{4}$)(13)	563.7($\frac{3,}{5}$)(16)	838.5($\frac{4,}{6}$)(20)	1174 ($\frac{5,}{7}$)(23)	4283 (1)(2)
局部失稳	123.6(2)(8)	274.5(3)(1)	486.1(4)(15)	759.2(5)(19)	1093 (6)(23)	

《表11》 $K_r = 1.5 \times 10^2$ $\mu_2 = 0.5$

	η_2	$\beta = 0.5$		$\beta = 1.5$		$\beta = 2.0$	
		$\chi_2 = -0.35$	$\chi_2 = +0.35$	$\chi_2 = -0.35$	$\chi_2 = +0.35$	$\chi_2 = -0.35$	$\chi_2 = +0.35$
总体失稳 $N = \infty$	1	11.00(1)(3)	14.58(1)(3)	35.08(1)(5)	43.21(1)(5)	47.87(1)(5)	56.97(1)(5)
	2	20.00(1)(3)	23.58(1)(3)	53.64(1)(4)	62.82(1)(4)	72.87(1)(5)	81.97(1)(5)
	5	35.50(1)(2)	42.14(1)(3)	101.6(1)(4)	110.8(1)(4)	137.8 (1)(4)	143.9 (1)(4)
	10	55.50(1)(2)	62.14(1)(2)	181.6(1)(4)	187.8(1)(3)	217.8 (1)(4)	223.9 (1)(4)
	20	95.50(1)(2)	102.1 (1)(2)	271.7(1)(3)	277.8(1)(3)	377.8 (1)(4)	383.9 (1)(4)
	50	215.5 (1)(2)	222.1 (1)(2)	541.7(1)(3)	547.8(1)(3)	675.3 (1)(3)	665.9 (1)(3)
	100	415.5 (1)(2)	422.1 (1)(2)	991.7(1)(3)	997.8(1)(3)	1125 (1)(3)	1115 (1)(3)
	200	815.5 (1)(2)	822.1 (1)(2)	1466 (1)(2)	1444 (1)(2)	2025 (1)(3)	2015 (1)(3)
	400	1195 (1)(1)	1201 (1)(1)	2244 (1)(2)	2244 (1)(2)	2943 (1)(2)	2871 (1)(2)
	1000	1795 (1)(1)	1801 (1)(1)	4666 (1)(2)	4644 (1)(2)	5343 (1)(2)	5271 (1)(2)
局部失稳	N = 1	22.36(2)(4)		78.36(2)(6)		110.8(2)(7)	
	N = 2	35.12(3)(5)		128.8(3)(8)		192.2(3)(9)	
	N = 3	48.78(4)(5)		192.2(4)(9)		299.2(4)(10)	
	N = 4	61.68(5)(6)		269.9(5)(10)		436.6(5)(12)	
	N = 5	78.36(6)(6)		363.6(6)(11)		604.6(6)(13)	

表中给出K_P值。括弧中的数值是失稳时的 波型(m)、(n)。(m)在前,(n)在后。(m)中所给出的是失稳时的主要纵向波数。

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