

复杂网络及其新近研究进展简介*

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摘要 简单地介绍了近 10 年来蓬勃发展的复杂网络研究新领域, 特别是其中最具代表性的随机网络、小世界网络和无尺度网络模型及其基本参数和概念, 并简要地回顾了复杂网络理论在生物和大脑神经网络、流行病传播与免疫控制、交通与社会经济、无线通讯、计算机和互联网、传感器网络、语言学与社会科学以及群体活动和编队飞行问题中的分散控制、稳定性特别是一致性问题等方面的应用趋势.

关键词 复杂网络, 规则网络, 随机网络, 小世界网络, 无尺度网络

1 引 言

20 世纪 80 年代以来, 以互联网为代表的计算机和信息技术的迅猛发展使人类社会大步迈入了一个“网络时代”. 从互联网到万维网、从电力网到交通网、从生物体中的大脑神经网到新陈代谢网、从科研合作网到各种政治、经济、社会关系网等, 人们事实上已经生活在一个充满着各种各样的复杂网络的世界之中. 这种全球网络化一方面极大地提高了人类的生产效率和生活质素, 如无线手机通话和电子邮件通信以及 GPS 交通导航等, 另一方面也给人类生活和社会活动带来了不少新的麻烦, 如传染病和计算机病毒的快速传播以及大面积停电事故等. 因此, 人类社会的日益网络化要求人类对各种人工和自然的复杂网络的结构、性质和行为有更清楚的认识和驾驭. 长期以来, 通信网络、电力网络、生物网络和社会网络等分别是通信科学、电力科学、生命科学和社会科学互不相交的研究对象, 而今天被称为复杂网络 (complex network) 的理论分支所包揽. 大体上说, 复杂网络要研究的是各种看上去互不相干但其实密切关联的形形色色网络之间的共同属性和处理它们的普适性方法. 从 20 世纪 90 年代开始, 复杂网络的研究逐步形成了一个自我完备的学科, 甚

至被称为“网络的新科学”^[1,2]. 复杂网络作为一个新的研究领域, 它的基本理论正渗透到从数理科学到生命科学、工程科学甚至社会科学等众多不同的领域中去. 事实上, 复杂网络的研究已经成为了近 10 年来全世界在不同学科领域 (包括力学、物理、生物、系统控制、通信技术、社会、经济和军事等) 中的科学家们的研究热点 (图 1). 今天, 对复杂网络的定性与定量特征的认识和理解已成为网络时代科学研究中一个共同而又极其重要的挑战性课题.

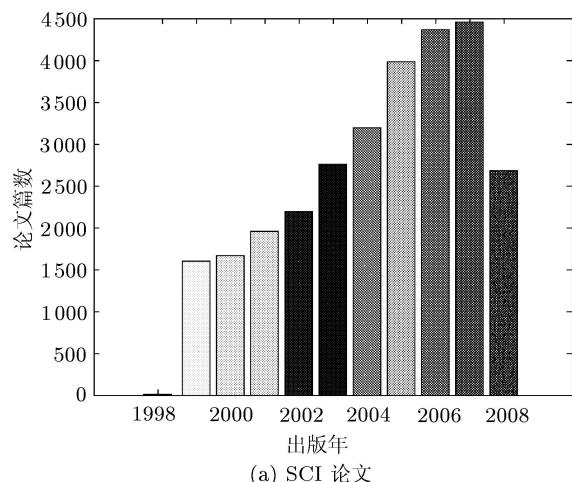


图 1 以“复杂网络”为关键词的 SCI 论文和 EI 论文发表情况统计

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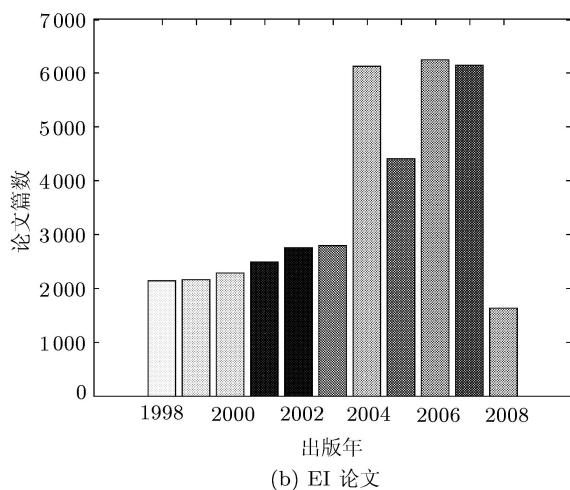


图 1 以“复杂网络”为关键词的 SCI 论文和 EI 论文发表情况统计 (续)

2 复杂网络的主要参数和基本模型

从数学的角度来说,一个网络可以用一幅图来表示。每幅图或网络都是由两个基本元素组成的:节点和边,其中一个节点可以代表互联网上的一台计算机、万维网上的一个网站、移动通信网中的一部手机、神经网络中的一个细胞、社会网络中的一个人或者一个公司等等,而一条边可以代表互联网上的一条光纤、万维网上的一种通路、移动通信网中的一次会话、神经网络中的一根经络、社会网络中的一层关系或者一笔交易等等。一个网络或者一幅图,由于节点和边之间存在不同的联结方式,就具有不同的拓扑结构并且可以具有非常不同的特性和行为,因而需要用不同的数学模型来描述^[3]。最具代表性的网络模型包括规则网络、随机网络、小世界网络和无尺度网络;各种网络中最重要的一些性质和参数包括度和度分布、群集系数、距离和平均最短路径长度、介数等^[4,5]。下面先定义复杂网络的几个主要参数、然后介绍相关的几个基本网络模型。

2.1 主要参数

度 (degree) 和度分布 (degree distribution): 网络上一个节点的度通常定义为这个节点具有的连接边的数目。例如,有 5 条连接边的节点的度就是 5。这样,把具有相同度的节点的个数分别统计起来,可以得到一张度的分布图,就像学生的成绩分布图一样。不同结构性质的网络具有不同的节点度分布,例如一般来说规则网络具有 δ 分布、随机

网络和小世界网络具有 Poisson 分布、无尺度网络具有幂指数分布。具有相同性质的不同网络会有相同的节点度分布,但它们节点度分布中的参数值会不尽相同。

群集系数 (clustering coefficient): 群集系数的概念用来刻画所关心的某个节点它的直接邻居节点之间也互相连接的稠密程度,定义为 0 到 1 之间的一个实数值。例如,假定某节点 A 的度为 5,即它有 5 个邻居节点;如果这 5 个邻居都互不相连,则节点 A 的群集系数为 0,但如果这 5 个邻居全都互相连接,则节点 A 的群集系数为 1。整个网络的群集系数定义为所有节点群集系数的平均值。显然星形规则网络的群集系数为 0,而全连接的规则网络的群集系数为 1。全连接网络和小世界网络都具有比较大的平均群集系数,随机网络和无尺度网络则具有比较小的平均群集系数。

距离 (distance) 和平均最短路径长度 (average shortest path-length): 两个节点之间的距离可以有许多不同的定义,其中最简单也最常用的就是它们之间最短连边的条数。例如,假定节点 A 和节点 B 直接相连,那么它们的距离是 1;如果它们连接的最短路径要通过另一个节点 C,那么它们的距离就是 2。一个网络的平均最短路径长度则定义为该网络上所有存在的节点距离的平均值。一般来说,全连接网络、小世界网络和随机网络都有比较短的平均最短路径长度,而链形、环形和树形等规则网络都具有比较长的平均最短路径长度。

介数 (betweenness): 介数分节点介数和连边介数,前者定义为网络中两两相连的节点对之间通过该节点的所有连边的总数量;后者定义为网络中两两相连节点对之间通过该边的所有连边的总数量。从概念上来说,介数可以理解为在网络上通过所关心的节点或者连边的数据流通量。显然,在讨论网络数据流通管理控制时,节点介数比节点度常常更为重要;不过节点度非常容易计算,介数就难算得多了。一个网络的节点(连边)平均介数就是网络上所有节点(连边)介数的平均值。一般来说,稀疏的网络介数低、稠密的网络介数高。

2.2 网络模型

要对不同的复杂网络进行描述和研究,首先要对它们建立相应的数学模型。对网络模型作严格分类是相当困难的。但大致上复杂网络可以划分为三大类型:首先是规则网络和随机网络;这

两种极端类型的网络能明确地定义, 其中规则网络包括全连接网络, 环形、链形、星形网络以及格点和分形图等非常规则的确定性网络, 而随机网络则指按照某种明确的统计规律生成的网络, 主要是经典的随机图模型及其派生出来的相关模型。其他一般性的网络模型只能笼统地归到第 3 类了, 是介于规则和随机网络模型之间的、有一定的规则但也有一定的随机性的所有其他网络; 这是一个很大的类别, 其中包括相当重要的小世界网络和无标度网络、以及加权网络和演化网络等等。本文以下部分只谈及规则网络、随机网络、小世界网络和无标度网络。

2.2.1 规则网络

规则网络 (regular network) 是最简单、但也是研究历史最长的一类网络, 可以追溯到 1736 年欧拉的七桥问题 [3,4], 甚至更早。除了像全连接网络, 环形、链形、星形网络以及格点和分形图等非常规则的确定性网络之外, 一种常见的规则网络是由多个节点组成的广义环状网络, 其中每个节点只与它最邻近的若干个节点连接。在规则网络中, 通常节点的度分布为 δ 函数, 节点的群集系数比较高, 平均最短路径长度也比较长。

2.2.2 随机网络

随机图 (random graph) 的数学理论始于 1960 年 Erdos 和 Renyi 的一篇经典论文 [6], 其后被称为 ER 随机图论, 应用于网络则称为随机网络模型, 定义为 [7]: 在由 n 个节点、总共 $C_n^2 = n(n-1)/2$ 条可能存在的边中, 随机地选取 m 条边所形成的网络, 记之为 $G(n, m)$ 。由这样的 n 个节点和 m 条边组成的网络共有 $C_{n(n-1)/2}^m$ 种, 全体构成一个概率空间, 每一个网络在其中出现的概率是相等的。另一种等价的随机网络模型是二项式模型, 定义为 [7]: 给定 n 个节点, 假定任意一对节点之间存在一条边的概率为 p , 记这样形成的网络为 $G_{n,p}$ 。整个网络中边的数目是一个随机变量, 其期望值为 $n(n-1)p/2$ 。因此得到一个 n 个节点和 m 条边的网络的概率为 $p^m(1-p)^{n(n-1)/2-m}$ 。如果令 $m = n(n-1)p/2$, 则两个模型 $G(n, m)$ 和 $G_{n,p}$ 等价。

ER 随机网络的节点度服从 Poisson 分布, 因而比较“均匀”即大部分的节点度相差不多, 并且它具有较小的平均最短路径长度和较小的群集系数。ER 随机网络模型在提出后、直到 20 世纪 90

年代末, 在近 40 年时间里逐步发展成了一门博大精深的数学分支, 衍生出不少变种, 也成为了描述各种复杂网络的主要工具。

20 世纪 90 年代以来, 由于现代大型高速计算机数据处理和运算能力的飞速发展, 人们对各种复杂网络进行了大规模的仿真试验, 结果发现现实世界中绝大多数的网络既不是完全规则的、又不是完全随机, 于是提出了一些更符合实际的网络模型, 其中小世界网络和无标度网络模型十分成功地描述了许许多多像互联网、万维网、无线通信网、电网、生物神经网、交通网、社会关系网、科研合作网以及与力学特别是统计力学和量子力学密切相关的形形色色的复杂网络。

2.2.3 小世界网络

小世界网络 (small-world network) 模型于 1998 年由美国 Cornell 大学理论和应用力学系的博士生 Watts 及其导师 Strogatz 提出 [8]。他们提出的 (WS) 小世界网络模型可以从一个具有 n 个节点的规则环形网络开始, 假定环上每一个节点都与两侧各 K 条边相连, 然后对每条边以概率 p 作随机重新连接, 即断开以后与随机选取的第 3 个节点相连 (但不允许自我连接和重复连接)。因为随机选取的第 3 个节点很可能位于远处 (这时, 新连接的边叫做“长程边”), 这种长程边的随机连接的结果就大大地减小了整个网络的平均最短路径长度, 而对网络的群集系数影响却不大, 使得生成的新网络具有所谓的“小世界”特性, 即同时具有较小的平均最短路径长度和较大的群集系数, 后者明显有别于 ER 随机网络模型。小世界网络也是比较“均匀”的, 即大部分的节点度相差不多, 节点度近似服从 Poisson 分布。

稍后, Newman 和 Watts 对上述的 WS 模型作了少许改动, 提出了另一个相近但较好的 (NW) 小世界网络模型 [9], 其做法是不去断开原来环形初始网络的任何一条边、而只是在随机选取的节点对之间增加一条边 (这时, 新连接的边很可能是长程边)。这一模型比 WS 模型容易分析, 因为它在形成过程中不会出现孤立的节点簇 (子图)。

值得提及的两个重要的相关工作是, Dorogovtsev 和 Mendes 对 NW 小世界网络模型作了精确求解 [10], 而 Kleinberg 则在二维格点图的基础上提出了一个一般性的小世界模型, 使得新模型的平均最短路径长度是可调的, 进而得到了一种

非常有效的快速算法^[11].

2.2.4 无标度网络模型

1999 年, 美国 Norte Dame 大学物理系的 Barabasi 教授和他的博士生 Albert 从统计物理的观点出发, 通过追踪万维网的动态演化过程, 发现了许多复杂网络都具有的一种大规模的高度自组织特性, 其中网络的节点度服从幂律分布: k^{-r} , 其中 k 是节点度变量而 r 是一个与标度无关的常数^[12]. 形成这种无标度网络 (scale-free network) 的两个要点是节点增加和择优连接, 特别是后者. Barabasi-Albert (BA) 无标度网络模型在生成过程中, 先从一个全连接的小网络 (最简单情形就是单个节点) 开始, 每一步以某种概率增加一个新的节点, 而新节点要与网络中已经存在的一些节点进行随机连接, 连接的概率和被选节点的度成正比 (称为择优连接). 可以证明, BA 网络的节点度服从 $r = 3$ 的幂律分布. 稍后, Albert 和 Barabasi 在 BA 模型中引进了小世界的那种随机重新连接边的机制, 得到了推广的 BA 模型, 允许其节点度服从 $2 < r < 3$ 的幂律分布^[13]. 后来, 其他研究人员还提出了许多变种的无尺度 BA 类型的模型^[4,5].

这些无尺度 BA 类型的模型的一个显著特点是它们结构的“不均匀”性, 即少数节点有很高的度、但大部分的节点的度却很小. 这种无标度网络的平均最短路径长度和群集系数也较小, 但比同规模的随机网络的群集系数要大.

复杂网络的小世界和无标度特性的发现是近年来网络研究的一次重要突破, 后来的许多现实网络的实证研究表明, 物理世界中的网络既不是非常理想的规则网络, 也不是完全随机的网络, 而是介乎于确定性与随机性之间的、具有某些特定规律同时具有某些统计特性的网络, 许多还兼容了小世界和无标度两者的一些基本性质. 10 年来, 复杂网络的研究开展得蓬蓬勃勃, 出版了不少专著^[1,4,5,14~20] 和许多综述性评论性文章^[21~29].

3 复杂网络研究的一些新进展

近 10 年来, 复杂网络已经成为了不同科学领域 (包括力学、物理、生物、系统控制、通信技术、社会、经济和军事等) 中的科学家们的热点研究方向. 随着研究的不断深入, 近二三年来复杂网络在诸多方面都取得了长足的进展, 如在复杂网络建模与拓扑结构^[30~41]、动力学性质^[18,42,43]、

网络涌现行为^[1,2,44,45]、复杂网络的鲁棒性与脆弱性^[45~49] 等方面.

在复杂网络群体行为和动力学的研究中, 同步和一致行为由于其重要的现实意义及其普遍性已经有很长时间的研究历史并且越来越受到广泛关注^[50~62]. 不同网络与不同类型的同步问题都受到了注意, 包括特殊耦合结构或特殊节点的网络同步^[63~80]、自适应同步行为^[81,82]、物理学中的相位同步与频率同步^[83~85]、有向网络同步^[86~91]、聚类同步^[92~99]、时滞网络同步或反同步^[100~106]、两个网络之间的同步^[107] 和全局同步^[108~110] 等等. 对于一个给定的复杂网络, 网络结构参数与同步能力有什么关系呢? 这一问题目前已有很多统计结果^[111~117]. 但是, 到目前为止, 尚未发现一个独立的结构参数能完全反映网络同步能力. 复杂网络同步控制, 特别是牵制控制 (pinning control) 也得到了深入研究^[118~125]. 与复杂网络同步问题直接相关的控制论问题, 尤其是群体活动 (swarming) 和编队飞行 (flocking) 问题中的分散控制、稳定性特别是一致性 (consensus) 问题, 近年来都受到了高度的重视^[126~157].

随着复杂网络理论研究的深入发展, 复杂网络应用方面的研究也得到了广泛关注, 特别是与生物体的新陈代谢系统、大脑神经网络相结合得到了大量的研究成果^[158~173], 与生物传染病相结合、在流行病传播与免疫控制方面取得了有意义的进展^[174~190]. 此外, 复杂网络上的博弈^[191~197]、复杂网络在交通网络与社会经济中的应用^[198~208]、通讯网络中的应用^[209~216]、计算机网与互联网中的应用^[217~221]、传感器网络中的应用^[222~235]、语言词汇网络与社会意见传播^[236~245] 等方面都有许多令人鼓舞的成果. 最近的一篇综述文献^[246] 和一期专刊^[247] 阐述了复杂网络多方面的实际应用.

以上提到的各个方面的理论和应用的研究充分反映了当前各个领域科研人员的共同兴趣以及他们对重要方向和课题的理解和认同, 值得国内同行们关注.

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INTRODUCTION TO COMPLEX NETWORKS AND THEIR RECENT ADVANCES*

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Abstract This article introduces the new notion of complex networks, especially the representative models of random-graph networks, small-world networks and scale-free networks as well as related basic concepts and modeling parameters. It also briefly summarizes their applications to biological systems and neural networks, epidemic spreading and immunization over networks, transportation systems, social and economic networks, wireless communication and sensor networks. And finally it reviews the problems of distributed control, stability and consensus analysis in their applications to swarming, flocking and synchronization.

Keywords complex network, random-graph network, small-world network, scale-free network

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